HEAT-CARRIER INTERACTION IN A THREE-FLOW HEAT EXCHANGER

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Analytical dependences are obtained for calculating the temperature fields of a three-flow heat exchanger, taking account of the nonlinearity of the balance relations.

In complex thermal-engineering systems (TS), multiflow heat exchangers are being used increasingly widely – in particular, three-flow heat exchangers [1]. The problem of correct description of the functional relation between the six boundary temperatures of such heat exchangers arises here (Fig. 1). In some problems, information is also required on the internal temperature profiles of each of the flows or it is at least required to determine the principal characteristic of this profile: the mean temperature value. The present work undertakes the investigation of these questions.

The methods traditionally used to solve this problem may be arbitrarily divided into three basic schemes: 1) numerical integration, for which the specification of three temperatures is required as the initial data; 2) artificial division of the three-flow heat exchanger into two two-flow units; 3) representation of the desired solution in symbolic form, with the aim of subsequent manipulation of the unwieldy matrices using a computer.

Each of these three schemes is relatively simple and convenient to use, but has serious deficiences, some of which would expediently be discussed here. For example, calculation by the first scheme requires significant machine time in comparison with analytical models. In fact, the number of divisions required to achieve technological accuracy (∿1%) is 50-200 steps. If the calculation algorithm for the system requires the specification not of end values but input flow temperatures, however, the problem is complicated by the need for an additional iterative process (for more details on the two-flow analog, see [2]).

In the second scheme, it is assumed that the inverse flow is divided between two direct flows, forming two parallel two-flow heat exchangers [3]. However, this equivalence is only possible in the case where the input temperatures of the direct flows are close, which is characteristic only for standard conditions. In the general case, however, the division coefficient of the inverse flow may vary markedly over the length of the heat exchanger. Thus, this calculation scheme may only be used in a narrow parameter range close to the standard conditions, i.e., is only suitable for design calculations and not for test calculations.

In the third scheme, general analytical solutions of matrix form characteristic of multiflow heat exchangers are constructed. This approach seems the most promising. However, nonlinearity of the problem leads to the need to organize the iterative process, i.e., to multiple manipulation of the matrices and considerable use of machine time. In addition, there is one other thermodynamic factor requiring specific form to be given to the general matrix relations. In fact, the efficiency of the heat exchanger is greatest, as a rule, when the water equivalents of the inverse flow and the sum of the direct flows are equal. The standard calculation conditions are usually based on these conditions. From a mathematical viewpoint, however, this condition leads to degeneracy of the matrix relations, i.e., the corresponding determinants vanish.

Thus, the optimal model which is of interest here must give an analytical relation between the temperatures in a compact form suitable for engineering calculations. The corresponding expressions are obtained below.

Consider the equations of a three-flow heat transfer, omitting the terms unimportant for the given problem: those associated with hydraulic losses, heat supply, heat conduction, etc. The influence of these factors on the heat transfer is assumed to be small, and may

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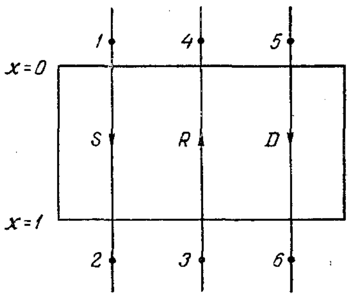

Fig. 1. Diagram of boundary-point indexing and conventional notation for three-flow heat exchanger.

be taken into account within the framework of perturbation theory for the given solution, as in the case of a two-flow heat exchanger [4]. Thus, fixing the flow rates and mean pressure in each flow, the following equations are obtained for static conditions in a counterflow heat exchanger

$$\begin{gathered}
G_S C_p^S \frac{\partial T_S}{\partial x} = (k\Pi)_S (T_R - T_S), \\
G_D C_p^D \frac{\partial T_D}{\partial x} = (k\Pi)_D (T_R - T_D), \\
-G_R C_p^R \frac{\partial T_R}{\partial x} = (k\Pi)_S (T_S - T_R) + (k\Pi)_D (T_D - T_R).
\end{gathered} \tag{1}$$

Suppose that the boundary temperatures for all the flow inputs are specified here: $T_S(0) = T_1$; $T_D(0) = T_5$; $T_R(1) = T_3$ (Fig. 1). The desired quantities are: the input temperatures (T_2, T_6, T_4); the temperature fields $T_S(x)$, $T_D(x)$, $T_R(x)$; and their mean values $\bar{T}_S$, $\bar{T}_D$, $\bar{T}_R$.

Two basic stages of the process may be isolated: 1) calculation of the roots f and g of the characteristic determinant of Eq. (1) – i.e., the exponents in the functions $T_i(x) = \alpha_i + \beta_i e^{fx} + \gamma_i e^{gx}$, i = S, R, D – and the relations between the preexponential coefficients; this stage simplifies the possibility of reducing Eq. (1) to a system of two equations in terms of $(T_S - T_R)$ and $(T_D - T_R)$; 2) introducing the basic functions in Eq. (7) (see Appendix), permitting the above-noted degeneracy to be obviated, and determining the coefficients of the expansion of $T_{S,D,R}$ with respect to these functions in specifying the boundary temperatures $T_{1,3,5}$. Finally, omitting the cumbersome intermediate calculations, the following expressions are obtained

$$\begin{gathered}
T_S(x) = T_1 + A_S(x)(T_3 - T_1) + B_S(x)(T_5 - T_1), \\
T_D(x) = T_5 + A_D(x)(T_3 - T_5) + B_D(x)(T_1 - T_5), \\
T_R(x) = T_3 + A_R(x)(T_1 - T_3) + B_R(x)(T_5 - T_3).
\end{gathered} \tag{2}$$

The functions $A_{S,D,R}(x)$ and $B_{S,D,R}(x)$ are determined by the interaction parameters in Eq. (1) and do not depend on the boundary tempratures (see Appendix). Of most interest are $A_S(1)$, $B_S(1)$, $A_D(1)$, $B_D(1)$, $A_R(0)$, $B_R(0)$, giving a set of output temperatures (T_2, T_6, T_4). They may be written in closed form

$$\begin{gathered}
A_S(1) = \frac{U_S}{D} [(1+b)F - b(1-a)G], \\
B_S(1) = \frac{U_S}{D} [b(G-F) + (1+ab)V_D FG], \\
A_D(1) = \frac{U_D}{D} [a(1+b)F + (1-a)G], \\
B_D(1) = \frac{U_D}{D} [a(G-F) + (1+ab)V_S FG],
\end{gathered} \tag{3}$$

$$A_R(0) = \frac{1}{D}[(f + U_S)F - a(g + U_D)G],$$

$$B_R(0) = \frac{1}{D}[b(f + U_S)F + (g + U_D)G];$$

$$D = V_S(1 + b)^2F + V_D(1 - a)^2 + 1 + ab,$$

$$F = [\exp(f) - 1]/f, \quad G = [\exp(g) - 1]/g,$$

$$f = c - e, \quad g = c + e, \quad e^2 = d^2 + V_SV_D,$$

$$a = -V_S/(d + e), \quad b = -V_D/(d + e),$$

$$C = \frac{1}{2}(V_D - U_D + V_S - U_S), \quad d = \frac{1}{2}(V_D - U_D - V_S + U_S), \tag{4}$$

$$U_S = \frac{(k\Pi)_S}{G_SC_p^S}, \quad U_D = \frac{(k\Pi)_D}{G_DC_p^D}, \quad V_S = \frac{(k\Pi)_S}{G_RC_p^R}, \quad V_D = \frac{(k\Pi)_D}{G_RC_p^R}$$

where the sign of e corresponds to the sign of d.

Analyzing Eqs. (3) and (4), the absence of zeros in the parameter D may be noted; this indicates complete lifting of the above-noted degeneracy. In fact, the result of equating the water equivalents within the framework of Eqs. (3) and (4) is that only f or g vanishes, which is easily dealt with by the expansions of the dependent functions F and G.

The temperature dependence of $C_p^{S,D,R}$ and $(k\Pi)_{S,D}$ is expediently taken into account by averaging these "constants" at each step of an iterative process specially organized within the framework of the given model. Here $(k\Pi)_{S,D}$ is best determined from the mean temperature values obtained from Eq. (2) with the substitution in Eq. (6) and the modification in Eq. (8). However, a different, traditional, method must be used in averaging $C_p^{S,D,R}$. In fact, the relation $G_SC_p^S(T_1 - T_2) + G_DC_p^D(T_5 - T_6) = G_RC_p^R(T_4 - T_3)$ following from Eqs. (3) and (4) must transform to the enthalpy-balance equation $G_S(h_1 - h_2) + G_D(h_5 - h_6) = G_R(h_4 - h_3)$ to bring this about, it is sufficient to assume that $C_p = (h_{out} - h_{in})/(T_{out} - T_{in})$ for each flow. If the calculation of large systems including three-flow heat exchangers requires an increase in speed, it is expedient to minimize the number of appeals to subprograms for calculating the heat-carrier enthalpy, by preliminary construction of the corresponding set of interpolation factors. However, experience in using the given programs shows that in actual TS not only linear but even cubic interpolation often introduces too much error into the calculation (up to 6% in regions of nonmonotonic behavior of C_p). This discrepancy is impermissible, since the whole refrigerating capacity of most TS is a few percent of the total heat transfer. The use of six-parameter interpolation is optimal in our view; it ensures an accuracy of within 0.5%. No more than three appeals to the thermodynamic program are required here, with simultaneous calculation of the local values of h and C_p at the ends of the temperature interval and in the center. In the normalized variables $x = (T - T_{min})/(T_{max} - T_{min})$; $y = (h - h_{min})/(h_{max} - h_{min})$, the corresponding polynomial may be written in the form

$$y(x) = (1 - a + b)x + (a - 3b + c - d)x^2 + (2b - 2c + 4d)x^3 + (c - 5d)x^4 + 2dx^5, \tag{5}$$

where the coefficients

$$a = \frac{1}{2}[y'(1) - y'(0)], \quad b = \frac{1}{2}[y'(0) + y'(1)] - 1,$$

$$c = 2\left[y'(1) - y'(0) + 8y\left(\frac{1}{2}\right) - 4\right],$$

$$d = 2\left[y'(0) + y'(1) + 4y'\left(\frac{1}{2}\right) - 6\right]$$

successively characterize the parabolic, cubic, fourth, and fifth degrees of nonlinearity.

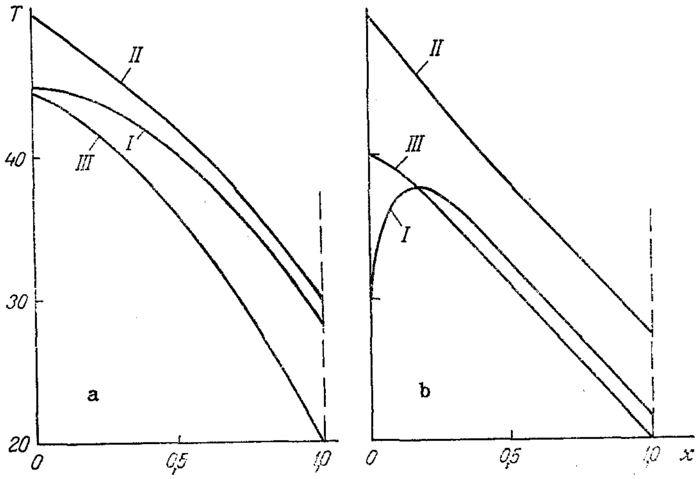

Fig. 2. Temperature profiles of a typical heat exchanger of cryogenic systems: a) normal profile; b) anomalous profile; I) direct flow S; II) direct flow D; III) inverse flow R.

Note that, in regions of monotonic variation in C_p, this approximation ensures an accuracy of hundredths of a percent. Some loss in accuracy may be due to the presence of a sharp maximum on the curve of $C_p(T)$ inside the temperature range of interest here. In the suprcritical region, the accuracy of the approximation remains within limits of 0.5%, as confirmed for $P > 2P_{cr}$.

In illustrating the relations obtained, note that, as well as the monotonic temperature profiles characteristic of standard conditions (Fig. 2a), various anomalous distributions which are of special interest in comparison with the two-flow situation are realized. In particular, two-flow heat transfer in analogous conditions does not permit intersection of the temperature profiles T_S and T_R. In the three-flow case, however, such intersection of the profiles is completely permissible (Fig. 2b) and is easily explained (If $G_S \ll G_R \sim G_D$, then T_S basically depends on the temperature T_R, the profile of which is determined by the interaction of T_R and T_D).

In conclusion, it is expedient to compare the calculation of the boundary temperatures with experimental data obtained in tests of one of the TS; the heat carrier is helium. Table 1 gives the set of experimental points and the set of theoretical values obtained analytically from Eqs. (2)-(5) (I) and by numerical inter integration with organization of the iterative process with fixing of the input temperatures (II). In the direct flow S, a flow rate of $1.72 \cdot 10^{-2}$ kg/sec and a pressure of 1.55 MPa is maintained here; in direct flow D, the corresponding values are $7.72 \cdot 10^{-2}$ and 0.87 and in inverse flow R, $8.96 \cdot 10^{-2}$ and 0.13. For these conditions, the variation in C_p on the flow path and the curvature of the profiles of $C_p(T)$ is sufficiently large; for example $C_p^{(2)} = 1.25\, C_p^{(1)}$; this, however, does not influence the accuracy of the calculations by the analytical model. It is clear from Table 1 that both calculation schemes give satisfactory agreement with experiment; the error is within 3% of the total temperature difference. As assumed above, the agreement with 1% accuracy between the two calculation methods requires 400 times more machine time in analytical scheme II than for calculation by analytical model I.

APPENDIX

The functions determining the temperature field in Eq. (2) take the form

$$A_S(x) = \frac{U_S}{D}\left[(1+b)\tilde{F}(x) - b(1-a)\tilde{G}(x)\right],$$

$$B_S(x) = \frac{U_S}{D}\left\{b\left[\tilde{G}(x) - \tilde{F}(x)\right] + (g+U_D)G\tilde{F}(x) + b(f+U_S)F\tilde{G}(x)\right\},$$

$$A_D(x) = \frac{U_D}{D}\left[a(1+b)\tilde{F}(x) + (1-a)\tilde{G}(x)\right], \tag{6}$$

TABLE 1. Comparison of the Results of Experiments and Calculations

Type of investigation	T_1	T_2	T_3	T_4	T_5	T_6
Experiment	34,4	Not measured	12,1	33,1	34,2	16,5
Calc.I	34,4	14,28	12,1	33,48	34,2	16,03
Calc.II	34,4	14,14	12,1	33,57	34,2	15,91

$$B_D(x) = \frac{U_D}{D}\{a[\tilde{G}(x) - \tilde{F}(x)] + (f + U_S)F\tilde{G}(x) - a(g + U_D)G\tilde{F}(x)\},$$

$$A_R(x) = \frac{1}{D}\{(f + U_S)[F - \tilde{F}(x)] + a(g + U_D)[\tilde{G}(x) - G] + (f + U_S)(g + U_D)[F\tilde{G}(x) - G\tilde{F}(x)]\},$$

$$B_R(x) = \frac{1}{D}\{b(f + U_S)[F - \tilde{F}(x)] + (g + U_D)[G - \tilde{G}(x)] + (f + U_S)(g + U_D)[G\tilde{F}(x) - F\tilde{G}(x)]\};$$

$$\tilde{F}(x) = [\exp(fx) - 1]/f, \quad \tilde{G}(x) = [\exp(gx) - 1]/g \tag{7}$$

where the other parameters correspond to the set in Eq. (4). The mean values of the functions $\bar{A}_{S,D,R}$ and $\bar{B}_{S,D,R}$ required to calculate the temperature $\bar{T}_2$, $\bar{T}_6$, $\bar{T}_4$ may be obtained by replacing $\tilde{F}(x)$ and $\tilde{G}(x)$ in Eq. (6) by

$$\bar{F} = \int_0^1 \tilde{F}(x)\,dx = (F - 1)/f \text{ and } \bar{G} = \int_0^1 \tilde{G}(x)\,dx = (G - 1)/g. \tag{8}$$

NOTATION

G, heat-carrier flow rate, kg/sec; T, temperature, K; h, specific enthalpy, J/kg; C_p, isobaric specific heat, J/kg·K; k, local heat-conduction coefficient, W/m^2·K; Π, heat-transfer surface, m^2; x, dimensionless coordinate, varying from 0 at one end of the heat exchanger (points 1, 4, 5) to 1 at the other (points 2, 3, 6; Fig. 1). Indices: S, first direct flow (input, point 1; output, point 2); D, second direct flow (input, point 5; output, point 6); R, inverse flow (input, point 3; output, point 4).

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